



第二届旋转与磁场下的 QCD 物质研讨会

1

Chaos in the near-horizon dynamics of the dyonic AdS_4 –Reissner–Nordström black hole

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Chaos in Strongly Coupled QCD Matter under Magnetic Field and Chemical Potential

Research Background

$$\Delta x(t) \sim e^{\lambda t}$$



Quantum Field Theories

$$\lambda \leq \frac{2\pi T}{\hbar}$$

J. Maldacena, S. H. Shenker and D. Stanford, JHEP 1608, 106 (2016).



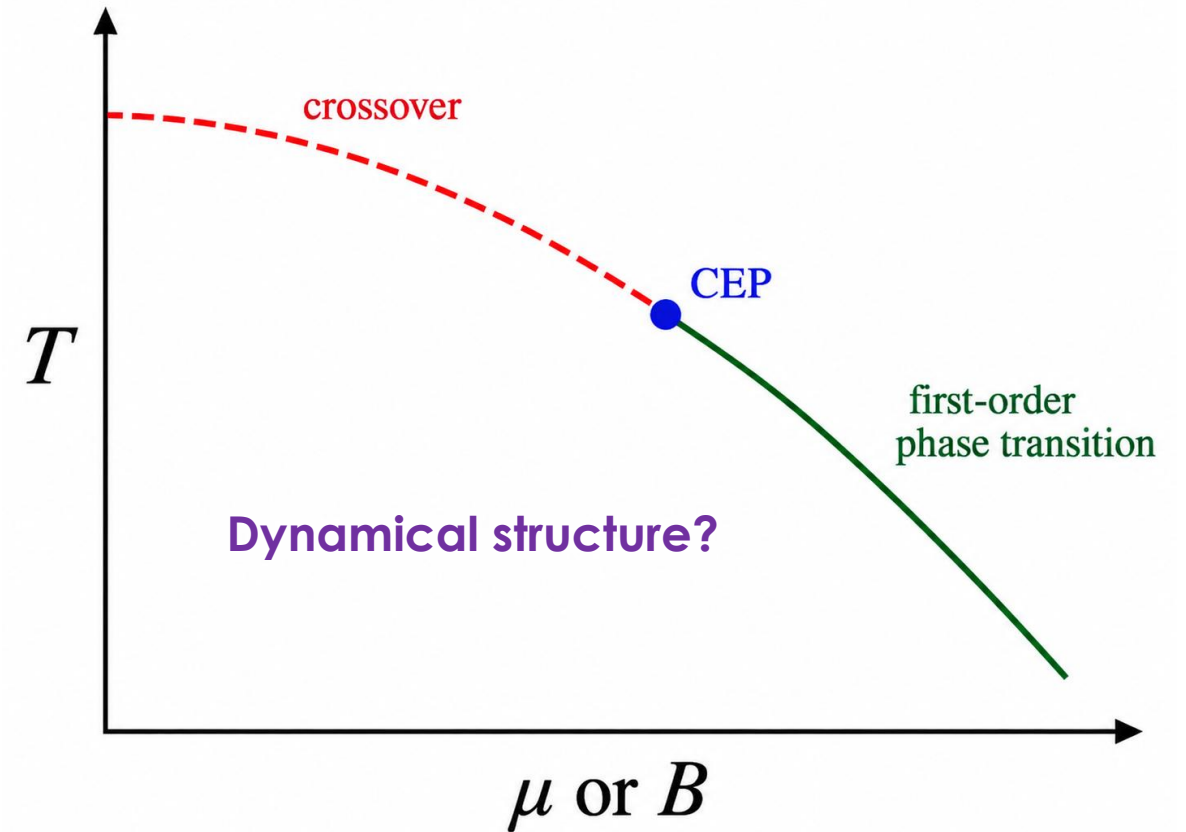
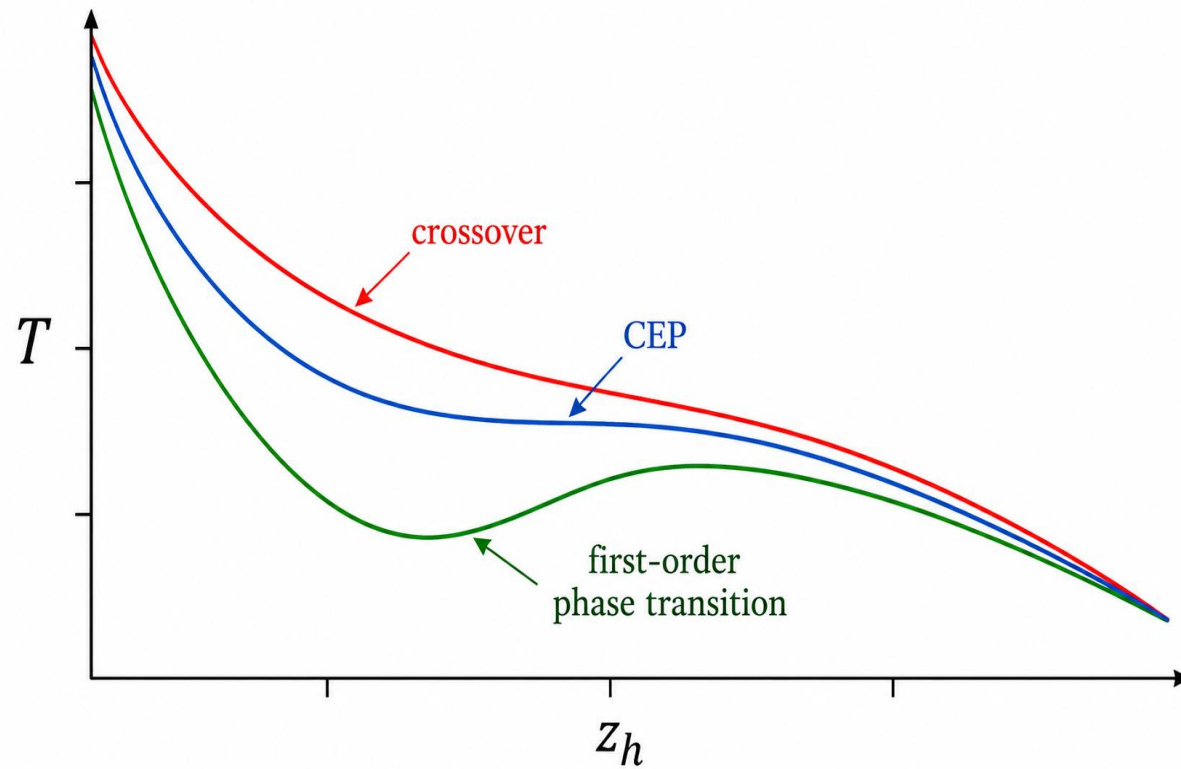
$$\lambda = \kappa$$

K. Hashimoto and N. Tanahashi, Phys. Rev. D 95 (2017) 024007

what provides the bound?

The horizon is a nest of the chaos.

Research Background



Dyonic AdS₄–Reissner–Nordström background

Metric:
$$ds^2 = \frac{1}{z^2} \left(-f(z) dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2 \right)$$

$$f(z) = 1 - \left(1 + \mu^2 z_h^2 + B^2 z_h^4 \right) \left(\frac{z}{z_h} \right)^3 + (\mu^2 z_h^2 + B^2 z_h^4) \left(\frac{z}{z_h} \right)^4$$

Electromagnetic field strength:
$$F = dA = -\frac{\mu}{z_h} dz \wedge dt + B dx \wedge dy$$

- $z = 0$: AdS boundary; $z = z_h$: outer horizon; $0 < z < z_h$: exterior region.

Temperature:
$$T = \frac{\kappa}{2\pi} = -\frac{f'(z_h)}{4\pi} = \frac{1}{4\pi z_h} \left(3 - z_h^2 (\mu^2 + z_h^2 B^2) \right)$$

Probe Dynamics

mass-shell condition: $g^{\mu\nu} p_\mu p_\nu = -m^2$

conserved energy for geodesic motion: $E = -\chi^\mu p_\mu = -p_t$

The timelike Killing vector: $\chi^\mu = (1, 0, 0, 0)$

Massless neutral probe: $m = 0$, $p_y = 0$: $E = \pm \sqrt{f(z) (f(z)p_z^2 + p_x^2)}$

The full Hamiltonian for an outgoing particle:

“+” : outgoing particle;
“−” : ingoing particle.

$$E = \sqrt{f(z) (f(z)p_z^2 + p_x^2)} + \frac{K_z}{2}(z - z_c)^2 + \frac{K_x}{2}(x - x_c)^2$$

Hamiltonian evolution

$$\begin{aligned}\dot{z} &= \frac{\partial E}{\partial p_z} = \frac{p_z f(z)^2}{\sqrt{f(z) (f(z)p_z^2 + p_x^2)}}, \\ \dot{p}_z &= -\frac{\partial E}{\partial z} = K_z (z_c - z) - \frac{(2f(z)p_z^2 + p_x^2) f'(z)}{2\sqrt{f(z) (f(z)p_z^2 + p_x^2)}}, \\ \dot{x} &= \frac{\partial E}{\partial p_x} = \frac{p_x f(z)}{\sqrt{f(z) (f(z)p_z^2 + p_x^2)}}, \\ \dot{p}_x &= -\frac{\partial E}{\partial x} = K_x (x_c - x).\end{aligned}$$

Classical observable in the probe model

$$\lambda_L = \lim_{t \rightarrow \infty} \lim_{\|\delta \mathbf{X}_0\| \rightarrow 0} \frac{1}{t} \ln \frac{\|\delta \mathbf{X}(t)\|}{\|\delta \mathbf{X}_0\|}$$

If $\lambda_L > 0$, the system exhibits chaotic motion;
if $\lambda_L = 0$, the motion is regular.

Gram–Schmidt orthogonalization

Toy model

Near-horizon radial toy model: $p_x = 0$ $V_{\text{harm}} = 0$

trajectory equation: $\dot{z} = \frac{\partial E}{\partial p_z} = f(z)$

blackening factor: $f(z) = 1 - (1 + \Gamma) \left(\frac{z}{z_h}\right)^3 + \Gamma \left(\frac{z}{z_h}\right)^4$ $\Gamma \equiv \mu^2 z_h^2 + B^2 z_h^4$.

➤ For nonextremal black hole $f(z) \approx -\frac{(3 - \Gamma)}{z_h} (z - z_h) + O((z - z_h)^2)$

Analytical solution: $z = z_h + c_1 e^{\frac{\lambda(\Gamma-3)}{z_h}}$ linear instability induced by the horizon.

➤ For extremal black hole limit: $f(z)|_{T=0} = 1 - 4 \left(\frac{z}{z_h}\right)^3 + 3 \left(\frac{z}{z_h}\right)^4 \approx \frac{6}{z_h^2} (z - z_h)^2 + \mathcal{O}((z - z_h)^3)$.

“softened” boundary condition: double zero

Analytical solution: $z = z_h + c_1$. radial instability vanishes

(μ, B) dependence of $f(z; \mu, B)$

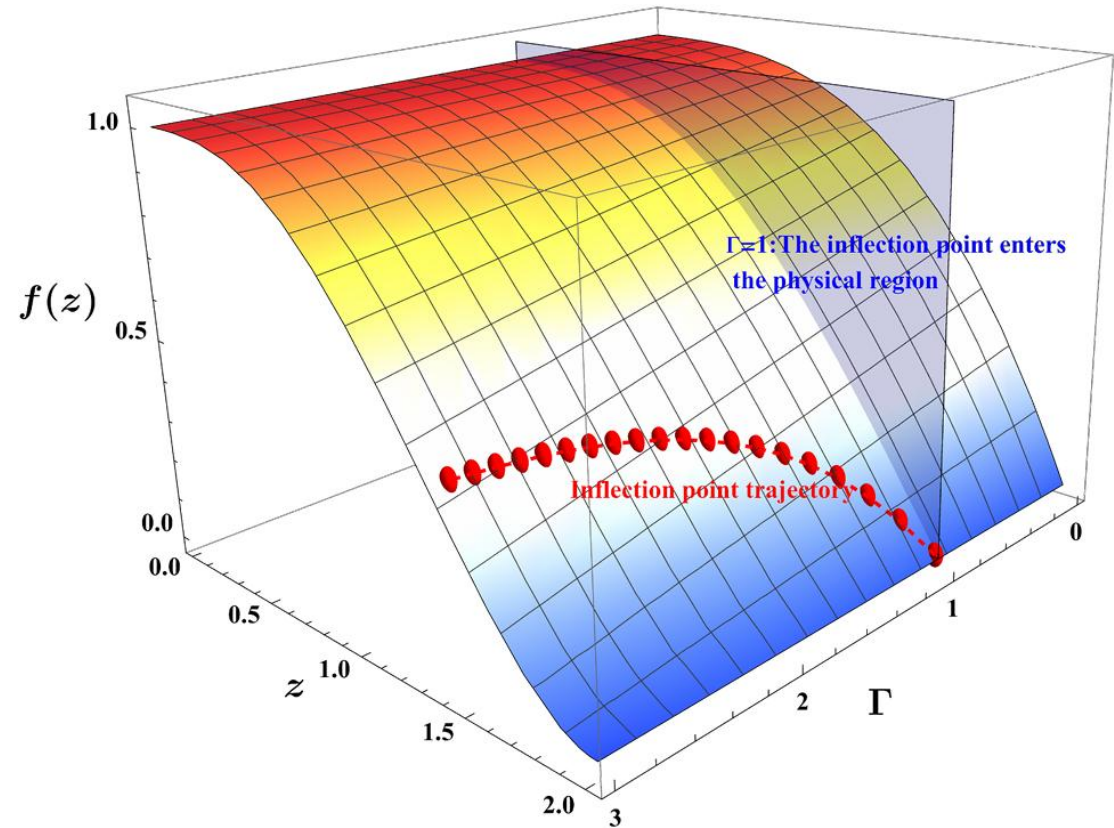
Γ alters $f(z)$ in two ways:

(1) Change in near-horizon slope:

$$f'(z_h) = -(3 - \Gamma)/z_h$$

(2) Distortion of the overall functional form:

$$z = \frac{z_h(1+\Gamma)}{2\Gamma} \text{ where } f''(z) = 0$$



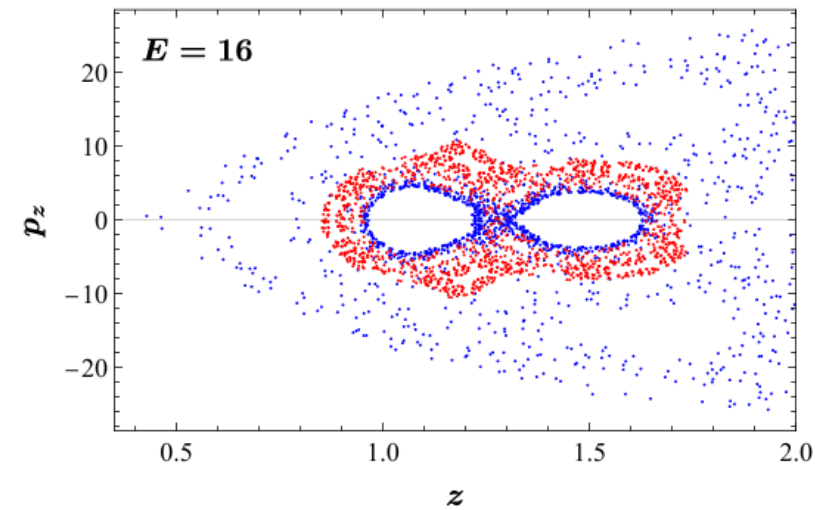
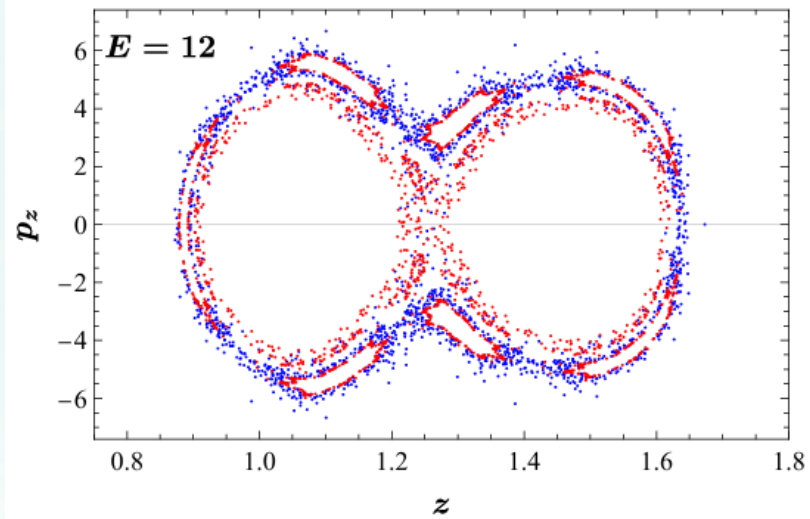
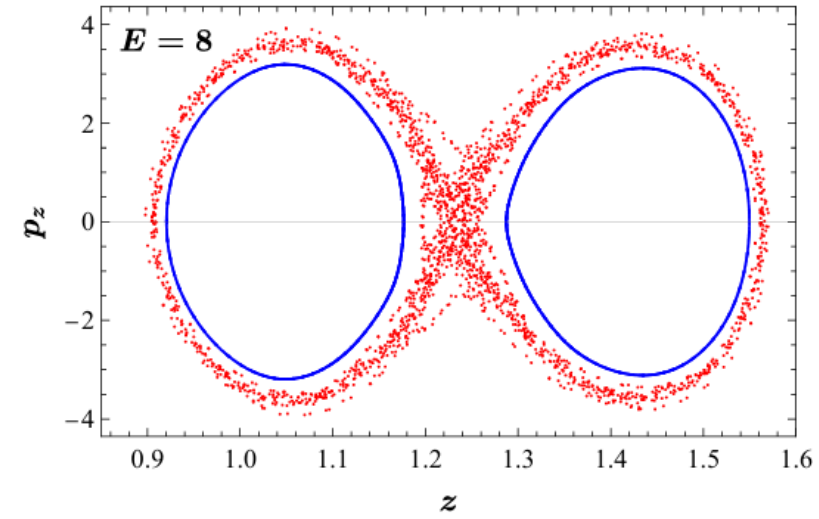
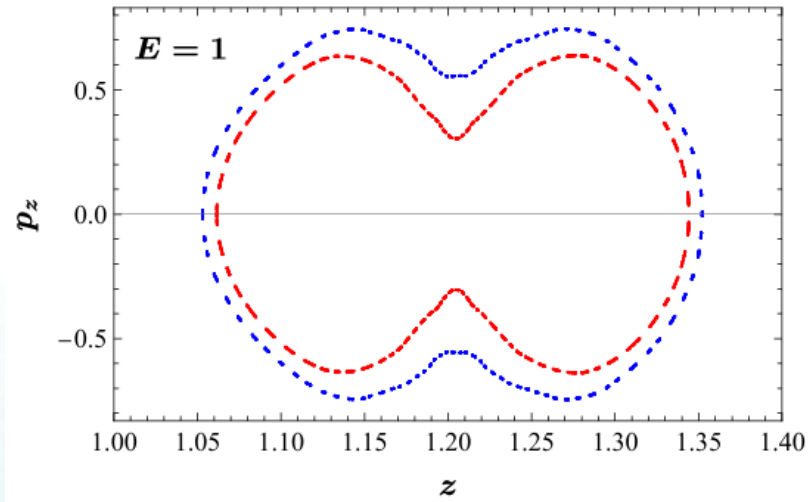
convex-concave structure

Numerical parameters and sampling prescription

Quantity	Value or prescription
Horizon and trap centre	$z_h = 2; z_c = 1.2; x_c = 0$
Confinement	$K_z = K_x = 50$
Initial condition I	$z = 1.2948; p_z = 0.7001$
Initial condition II	$z = 1.2741; p_z = 0.6391$
Section prescription	$x = 0; p_x > 0$ determined by $H = E$
Dynamical maps	$E = 3, 7, 11, 14; 4\mu^2 + 16B^2 \leq 3$

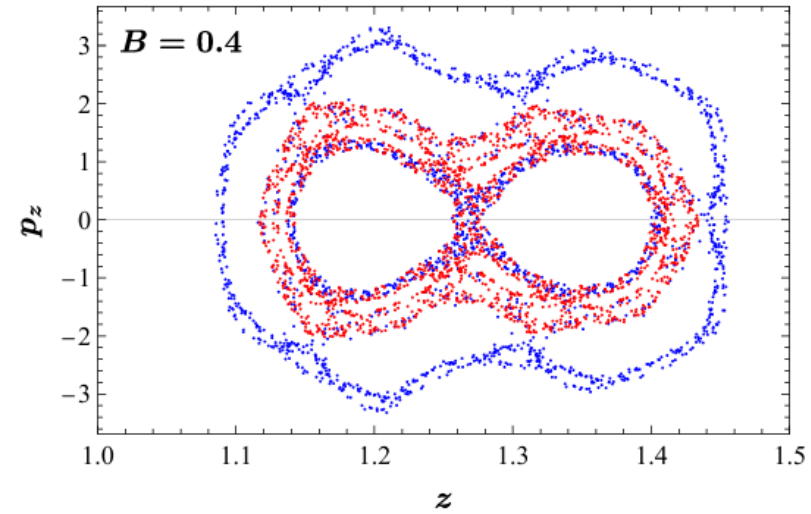
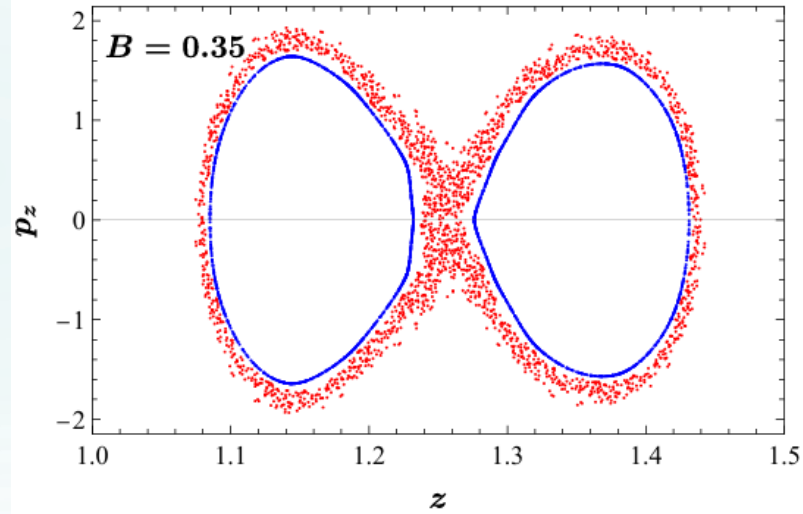
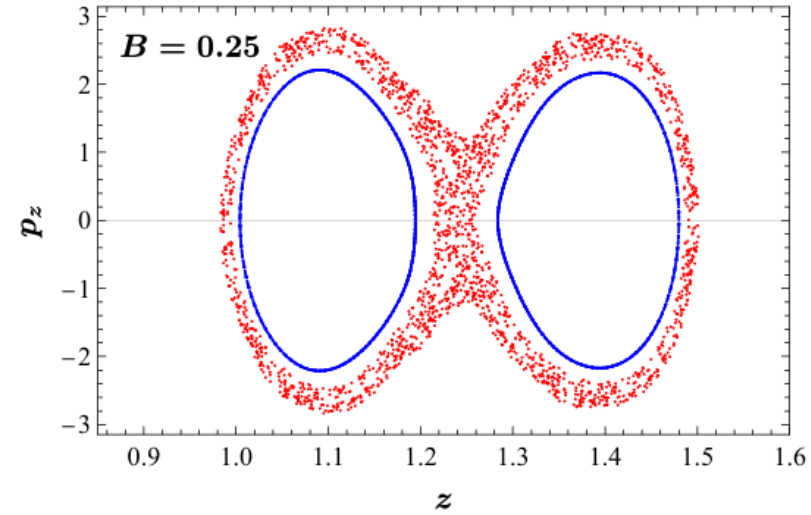
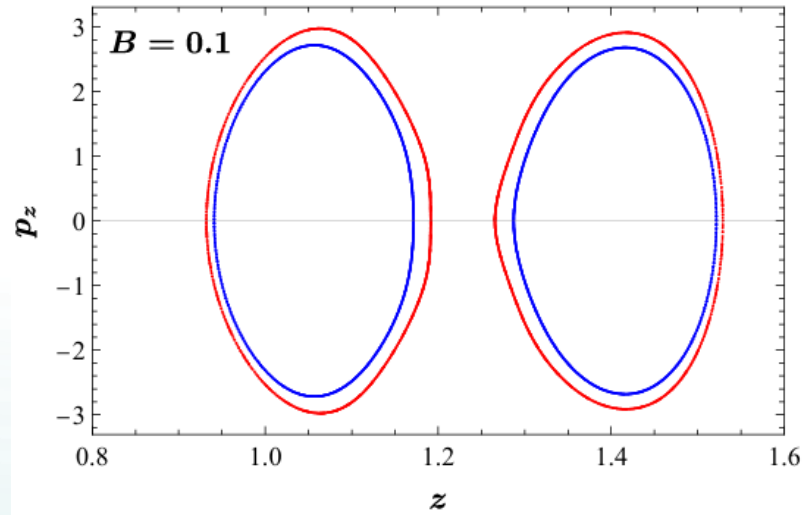
The section plots sample two initial conditions. Figure 5 presents finite-time Lyapunov histories for initial condition II, up to $t = 3000$.

Energy dependence of the Poincaré sections

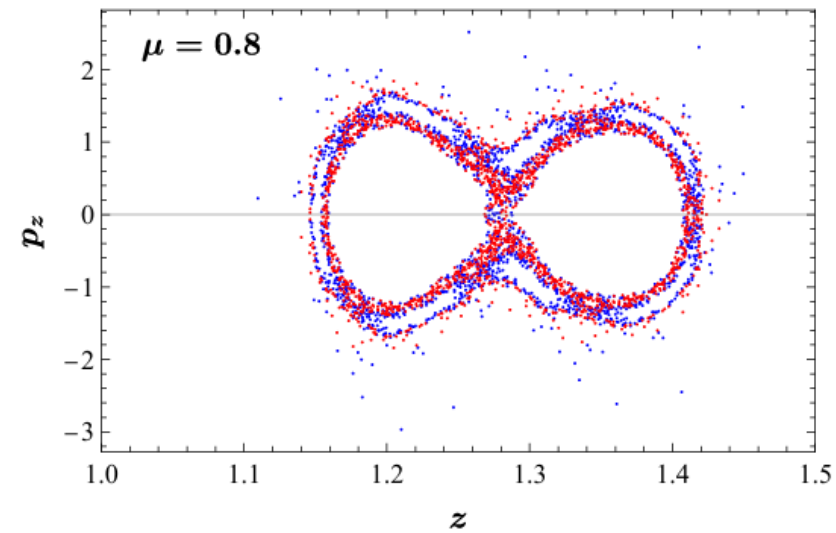
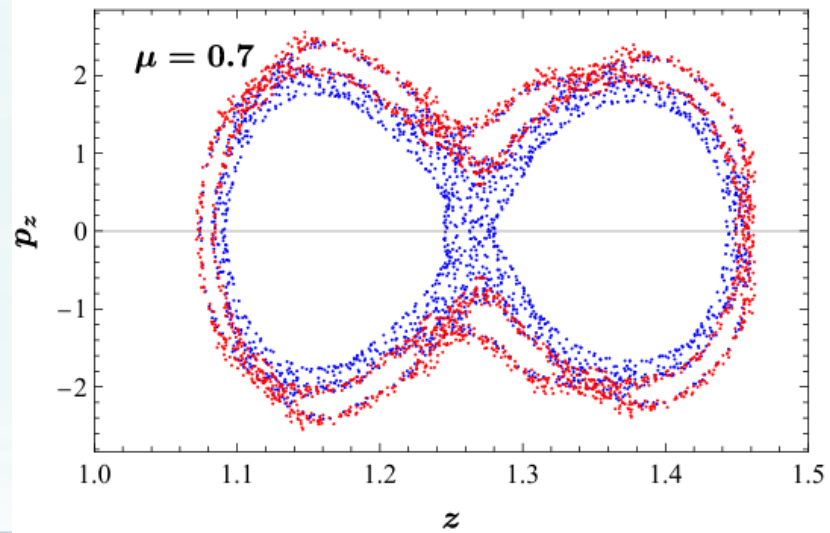
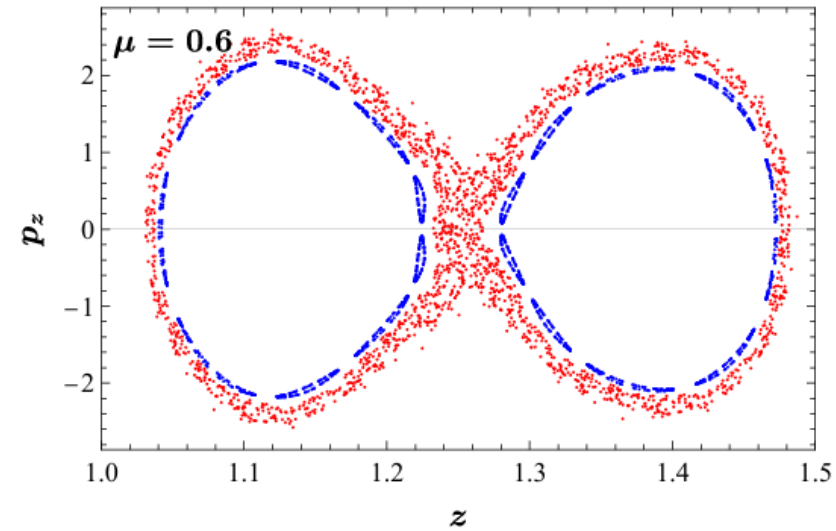
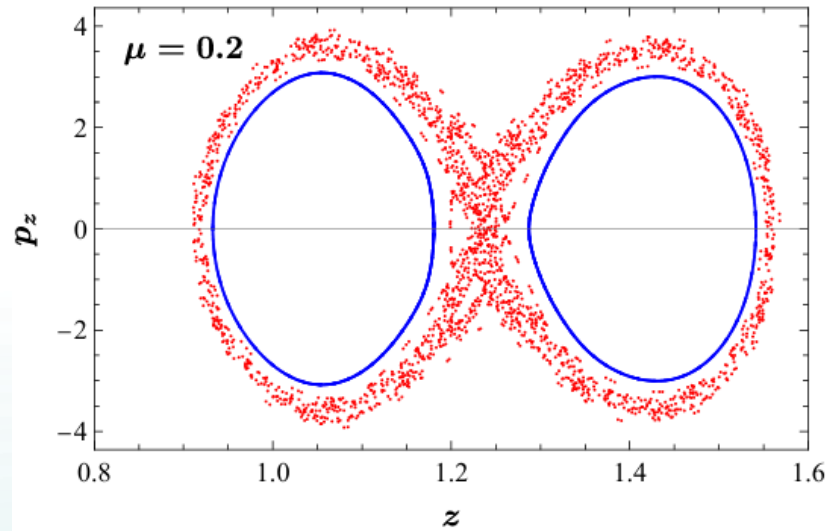


Magnetic-field dependence at fixed energy

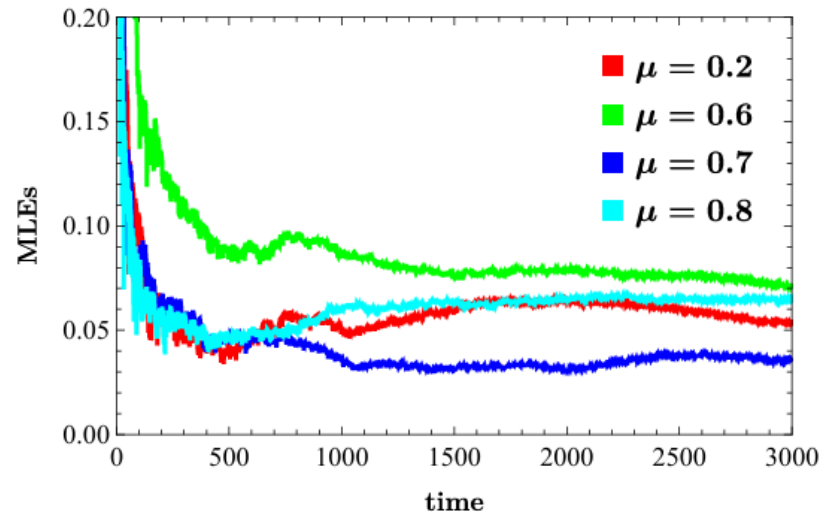
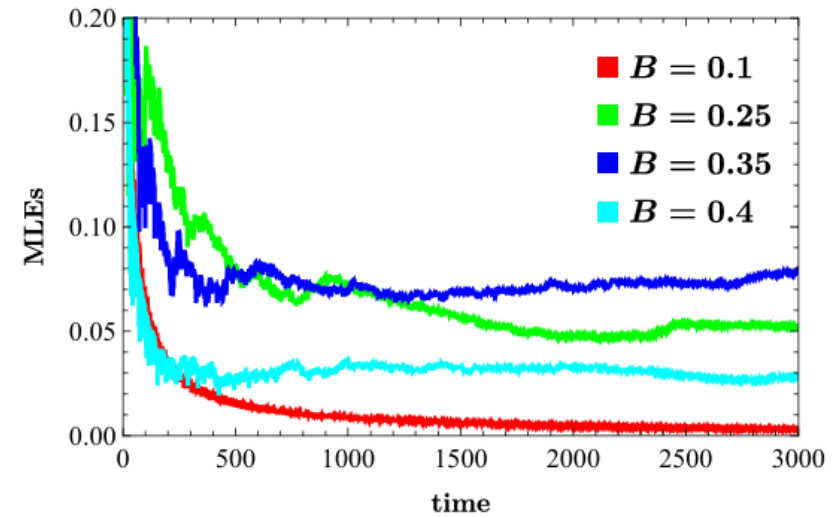
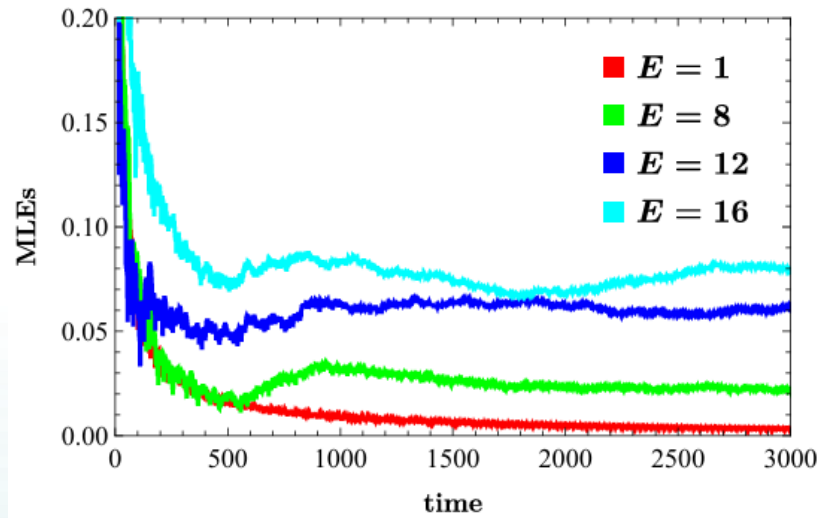
11



Chemical-potential dependence at fixed energy

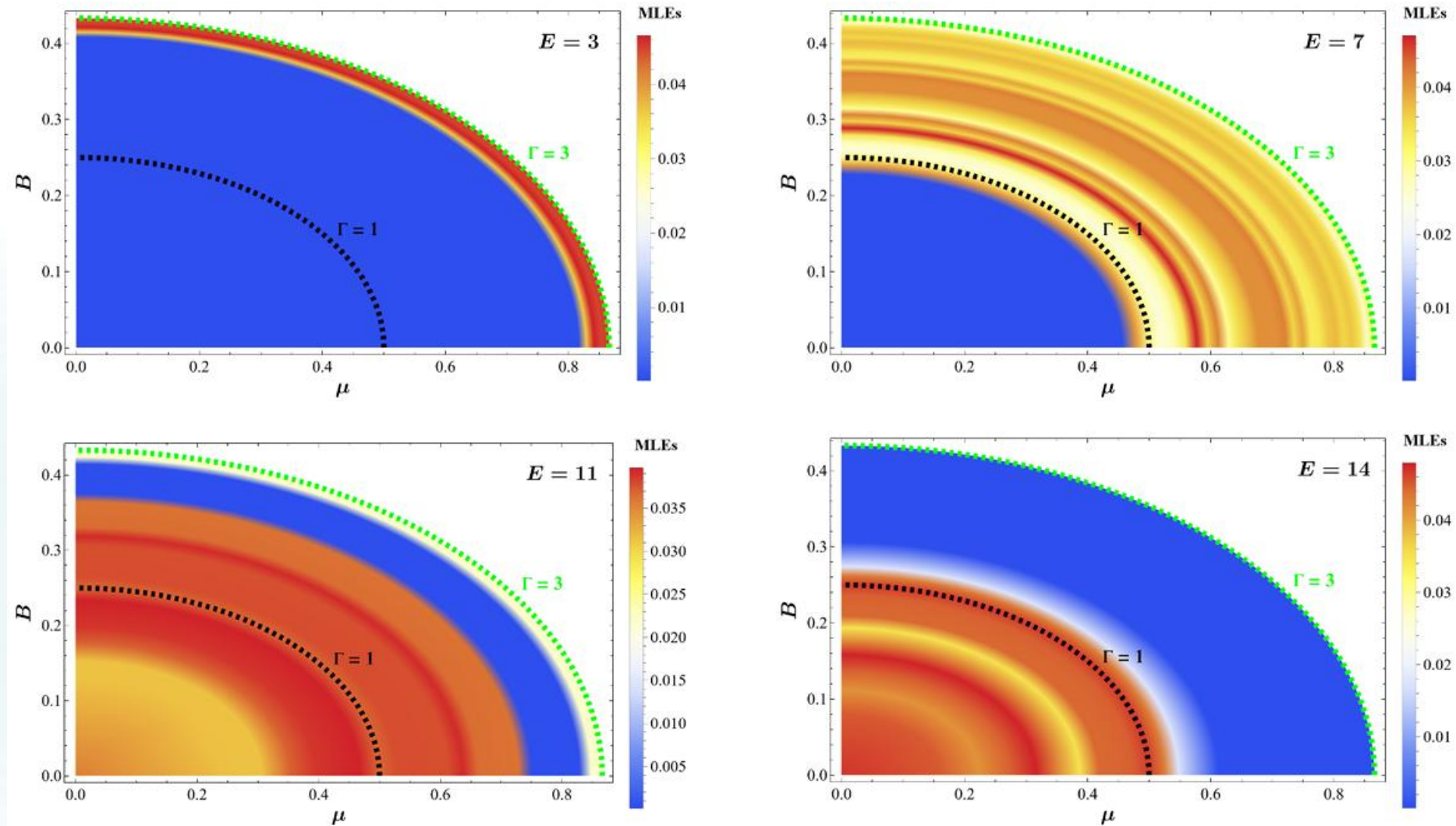


Finite-time maximum Lyapunov estimates



Dynamical maps in the (μ, B) parameter plane

14



Conclusions

- In a region relatively far from the event horizon of a black hole(corresponding to lower energy), increasing μ or B generally enhances chaos, fragmenting Kolmogorov-Arnold-Moser tori and expanding chaotic seas in phase space.
 - As the extremal limit ($\Gamma = 3$) is approached the Lyapunov exponent can remain finite and positive, thereby violating the upper bound $\lambda_L \leq \kappa$.
 - As the system approaches to the extremal black hole limit ($\Gamma = 3$), chaotic dynamics are quenched in a region very close to the event horizon of a black hole(corresponding to higher energy), giving rise to a corridor of regular motion in the (μ, B) plane.
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Dynamical implication for QCD dynamics

Holographic Result	Possible QCD Implication
B, μ enhance chaos and break KAM structures	Enhanced nonlinear fluctuations and information scrambling in strongly coupled QCD matter
λ_L exceeds the conventional bound near extremality	Faster nonequilibrium response under extreme conditions
Regular dynamical corridor emerges	Possible dynamically stabilized regions with suppressed fluctuations